PHY – Y

GS-2018-Y (Physics)

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Written Test in PHYSICS – December 10, 2017

Duration : 3 (Three) Hours

NAME:____ REF. CODE:__________________

PLEASE READ THESE INSTRUCTIONS CAREFULLY BEFORE ATTEMPTING THE QUESTIONS

1. Please fill in details about name, reference code etc. on the answer sheet and question paper. Use only blue/black ballpoint pen. The Answer Sheet is machine –readable and will not read other colours.
2. This test consists of three parts: **Section A**, **Section B** and **Section C**. You must answer questions according to the programme you are applying for.

Candidates applying for	**Must answer**	**Should not attempt**
Integrated M.Sc.-Ph.D.	Section **A** + Section **B**	Section **C**
Ph.D.	Section **A** + Section **C**	Section **B**

Section A has 20 questions : 1 – 10 are multiple-choice; 11 – 20 are numerical.

Section B has 15 questions : 21 – 30 are multiple-choice; 31 – 35 are symbolic.

Section C has 15 questions : 36 – 45 are multiple-choice; 45 – 50 are symbolic

3. Indicate your ANSWER ON THE OMR ANSWER SHEET as follows.

   Multiple choice questions have four options (a), (b), (c) and (d), of which only one option is correct. Indicate the answers by filling up the bubble on the Answer Sheet corresponding to the correct option. If more than one bubble is filled in, it will be treated as not answered.

   Numerical questions have answers which are 3 (three) digit integers. Indicate the answers by filling in the corresponding bubbles on the Answer Sheet. Unless all three bubbles for a given question are filled, it will be treated as not answered. (See inside for details.)

   Symbolic questions have answers which are a number, a short formula or a word. Indicate the answers by writing in the boxes on the Answer Sheet next to the appropriate question numbers. (See inside for details.)

4. The marking for these questions shall be as follows.

If the answer is	**Multiple-choice**	**Numerical**	**Symbolic**
Correct	+3	+5	+5
Incorrect	−1	0	0
Not attempted	0	0	0
Multiple options marked	0	—	—

   Note that only multiple-choice type questions have negative marking.

5. Candidates are advised to mark the Answer Sheet only when they are sure of the answer. Till then, they may mark the answers on the question paper.

continued on next page…

6. Rough work may be done on blank pages of the question paper. If needed, candidates may ask for extra sheets from the invigilators.
7. Use of scientific, non-programmable calculators is permitted. Calculators which plot graphs are NOT allowed. Multiple-use devices, such as cell phones, smartphones, etc. CANNOT be used as calculators.
8. Candidates should NOT ask the invigilators for clarifications regarding the questions. They have been instructed not to respond to any such queries. In case a correction/clarification is deemed necessary, it will be announced in the examination hall.
9. A list of useful physical constants is given on the next page. Make sure to use only these values in answering the questions, especially those of numeric type.

USEFUL CONSTANTS

Symbol	Name/Definition	Value (Units)
c	speed of light in vacuum	3×10^{8} m s^{-1}
$\hbar$	reduced Planck constant ($= h/2\pi$)	1.04×10^{-34} J s
G_N	gravitational constant	6.67×10^{-11} m^3 kg^{-1} s^{-2}
$M_\odot$	solar mass	1.989×10^{30} kg
ε_0	permittivity of free space	8.85×10^{-12} F m^{-1}
μ_0	permeability of free space	$4\pi \times 10^{-7}$ N A^{-2}
e	electron charge (magnitude)	1.6×10^{-19} C
m_e	electron mass	9.1×10^{-31} kg
		$= 0.5$ MeV/c^2
a_0	Bohr radius	0.51 Å
	ionisation potential of H atom	13.6 eV
N_A	Avogadro number	6.023×10^{23} mol^{-1}
k_B	Boltzmann constant	1.38×10^{-23} J K^{-1}
		$= 8.6173 \times 10^{-5}$ eV K^{-1}
$R = N_A k_B$	gas constant	8.31 J mol^{-1} K^{-1}
$\gamma = \frac{C_p}{C_V}$	ratio of specific heats: monatomic gas	1.67
	diatomic gas	1.40
σ	Stefan-Boltzmann constant	5.67×10^{-8} W m^{-2} K^{-4}
α	fine structure constant ($= e^2/4\pi\varepsilon_0\hbar c$)	$1/137$
g	acceleration due to gravity	9.8 m s^{-2}
R_E	radius of the Earth	6.4×10^{3} Km
R_S	radius of the Sun	7×10^{5} Km
m_p	proton mass ($\approx 2000\, m_e$)	1.7×10^{-27} kg
		$= 938.2$ MeV/c^2
m_n	neutron mass ($\approx 2000\, m_e$)	1.7×10^{-27} kg
		$= 939.6$ MeV/c^2

CONVERSIONS

Symbol	Name/Definition	Value (Units)
1 A.U.	mean distance of Earth from Sun	1.5×10^{9} km
1 a.m.u.	atomic mass unit	1.6×10^{-27} kg
		$= 931.5$ MeV/c^2
1 eV	electron Volt	1.6×10^{-19} J
1 T	Tesla	10^{4} gauss
1 bar	mean atmospheric pressure at 0^0 C	1.01×10^{5} Pa (= N m^{-2})
1 Å	Ångstrom unit	10^{-8} cm

GS-2018-Y (Physics)

Section A

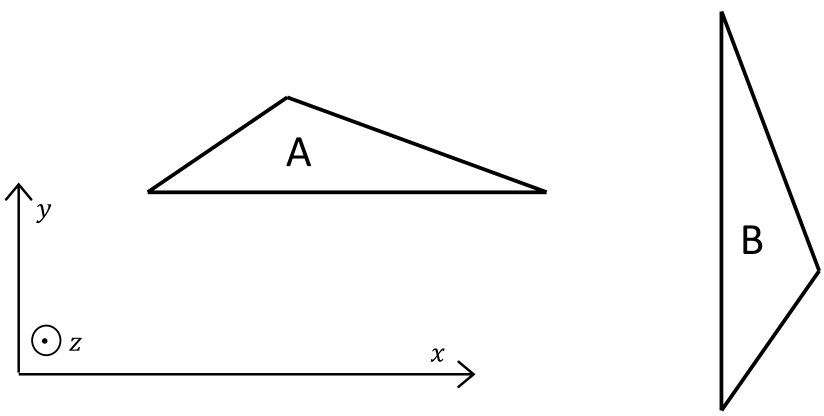
Q. 1 – 20 : to be attempted by ALL candidates

PLEASE READ CAREFULLY BEFORE PROCEEDING FURTHER

The following questions (1 – 10) are all of multiple-choice type. For every question, four options (a), (b), (c) and (d) are given, of which *only one* is correct. Indicate the correct option on the OMR by filling only one bubble. If more than one bubble is filled the question will be treated as not attempted.

3 marks will be awarded for each correct answer and 1 mark will be deducted for each incorrect answer. If the question is not attempted, no marks will be awarded or deducted.

1.



Refer to the figure above. If the z-axis points out of the plane of the paper towards you, the triangle marked 'A' can be transformed (and suitably re-positioned) to the triangle marked 'B' by

(a) rotation about z-direction by $\pi/2$, then reflection in the yz-plane
(b) reflection in the xz-plane, then rotation by $-\pi/2$ about z-direction
(c) reflection in the yz-plane, then rotation by $\pi/2$ about z-direction
(d) rotation about x-direction by $\pi/2$, then rotation by $-\pi/2$ in the yz-plane

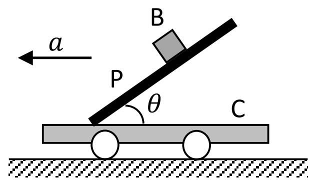
2. If a 2×2 matrix $\mathbb{M}$ is given by

$$\mathbb{M} = \begin{pmatrix} 1 & (1-i)/\sqrt{2} \\ (1+i)/\sqrt{2} & 0 \end{pmatrix}$$

then $\det \exp \mathbb{M} =$

(a) e^2 (b) e (c) $2i \sin\sqrt{2}$ (d) $\exp\left(-2\sqrt{2}\right)$

3. A small block B of mass m is placed on an inclined plane P, which makes an angle θ with a horizontal cart C, on which P is rigidly fixed (see figure). The coefficient of friction between the block B and the plane C is μ. When the cart stays stationary the block slides down. If the cart C is moving in the horizontal direction with acceleration a, the minimum value of a for which the block remains static on the plane is



(a) $g\dfrac{\tan\theta - \mu}{\mu\tan\theta + 1}$ (b) $g(\mu - \sin\theta\cos\theta)$

(c) $g\dfrac{1 - \mu\tan\theta}{\mu + \tan\theta}$ (d) $g\,(\cos\theta - \mu\sin\theta)$

4. A particle of mass m moving in one-dimension x is subjected to the Lagrangian

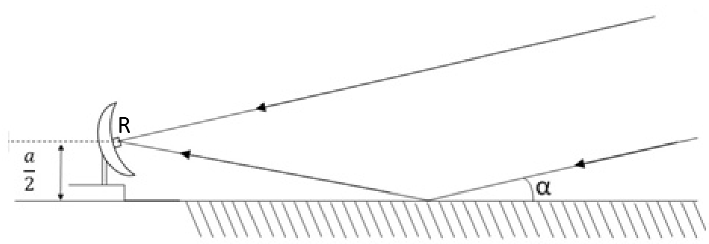
$$L = \frac{1}{2}m(\dot{x} - \lambda x)^2$$

where λ is a real constant. If it starts at the origin at $t = 0$, its motion corresponds to the equation (a is a constant)

(a) $x = a\exp\lambda t$ (b) $x = a\sin\lambda t$

(c) $x = a\{1 - \exp(-\lambda t)\}$ (d) $x = a\sinh\lambda t$

5. The sketch below shows a radio antenna located at the edge of a calm lake, which has a receiver R at the centre of the dish at a height $a/2$ above the ground. This is picking up a signal from a distant radio-emitting star which is just rising above the horizon. However, the receiver also picks up a reflected signal from the surface of the lake, which, at the relevant radio-wavelength, may be taken to be a plane.



If the instantaneous angle of the star above the horizon is denoted α, the receiver R will detect the first interference maximum when $\alpha =$

(a) $\arcsin\left(\dfrac{\lambda}{2a}\right)^{1/3}$ (b) $\arcsin\left(\dfrac{\lambda}{a}\right)^{1/3}$ (c) $\arcsin\dfrac{\lambda}{2a}$ (d) $\arcsin\dfrac{\lambda}{a}$

6. A particle is confined inside a one-dimensional box of length ℓ and left undisturbed for a long time. In the most general case, its wave-function MUST be

(a) the ground state of energy.
(b) periodic, where ℓ equals an integer number of periods.
(c) any one of the energy eigenfunctions.
(d) a linear superposition of the energy eigenfunctions.

7. A classical ideal gas of atoms with masses m is confined in a three-dimensional potential

$$V(x, y, z) = \frac{\lambda}{2}(x^2+y^2+z^2)$$

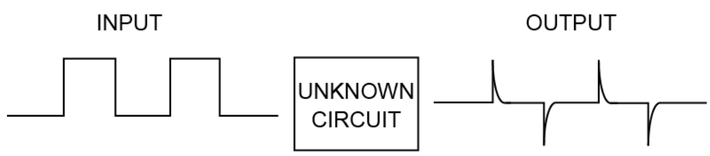
at a temperature T. If k_B is the Boltzmann constant, the root mean square (r.m.s.) distance of the atoms from the origin is

(a) $\left(\frac{3k_BT}{\lambda}\right)^{1/2}$ (b) $\left(\frac{2k_BT}{3\lambda}\right)^{1/2}$ (c) $\left(\frac{3k_BT}{2\lambda}\right)^{1/2}$ (d) $\left(\frac{k_BT}{\lambda}\right)^{1/2}$

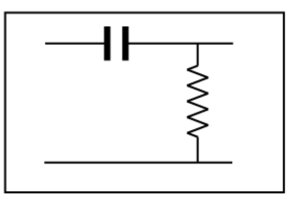
8. The characteristic impedance of a co-axial cable is independent of the

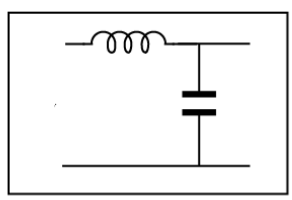
(a) dielectric medium between the core and the outer mesh
(b) length of the cable
(c) outer diameter
(d) core diameter

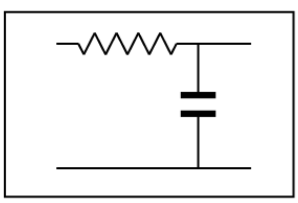
9. The figure below shows an unknown circuit, with an input and output voltage signal.

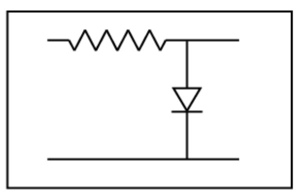


From the form of the input and output signals, one can infer that the circuit is likely to be

(a)


(b)


(c)


(d)


10. In Boolean terms, (A + B) (A + C) is equal to

(a) ABC
✔(b) A + BC
(c) A(B + C)
(d) (A + B + C) (A + B)

Section A continues...
(to be answered by ALL candidates)

PLEASE READ CAREFULLY BEFORE PROCEEDING FURTHER

The answers to the following questions (11 – 20) are all integers of 3 (three) digits each. You may round off decimal parts, e.g. $122.5 \leq x < 123.5$, as $x = 123$ and e.g. $123.5 \leq x < 124.5$ as $x = 124$ and so on. Use only values of constants given in the table 'USEFUL CONSTANTS'.

Answer these questions on the OMR by filling in bubbles as you did for your reference code. Note that if the answer is, e.g. 25, you must fill in 025 and if it is, e.g. 5, you must fill in 005. If it is 0, you must fill in 000. If these preceding zeros are not filled in (where required), the question will be treated as not attempted.

5 marks will be awarded for every correct answer. There are NO NEGATIVE MARKS for these questions.

11. Consider the two equations

$$\frac{x^2}{3} + \frac{y^2}{2} = 1$$

$$x^3 - y = 1$$

How many simultaneous real solutions does this pair of equations have ? **002**

12. A heavy steel ball is struck by a small steel pellet moving horizontally with velocity 20 m s^{-1}. If the pellet bounces off the steel ball with no slippage, and then rises vertically to a height 10 m above the point of contact, then what is 100 times the elastic coefficient of restitution (e) i.e. $100e$? **049**

13. A particle is in the ground state of a cubical box of side ℓ. Suddenly one side of the box changes from ℓ to 4ℓ. If p is the probability of finding the particle in the ground state of the new box, what is $1000p$? **058**

14. The wave-function Ψ of a particle in a one-dimensional harmonic oscillator potential is given by

$$\Psi = \left(\frac{1}{\pi\ell^2}\right)^{1/4}\left(1+\frac{\sqrt{2}x}{\ell}\right)\exp\left(-\frac{x^2}{2\ell^2}\right)$$

where $\ell = 100\ \mu$m. Find the expectation value of the position x of this particle, in μm. **071**

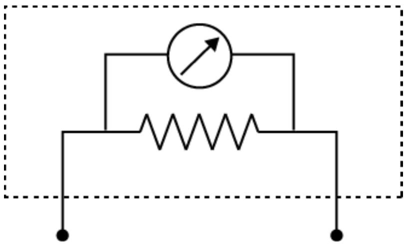
15. Consider a dipole antenna with length ℓ, charge q and frequency ω. The power emitted by the antenna at a large distance r is P. Now suppose the length ℓ is increased to $\sqrt{2}\ell$, the charge is increased to $\sqrt{3}q$ and the frequency is increased to $\sqrt{5}\omega$. By what factor is the radiated power increased ? **150**

16. Calculate the self-energy, in Joules, of a spherical conductor of radius 8.5 cm, which carries a charge 100 μC. **529**

17. A heat engine is operated between two bodies that are kept at constant pressure. The constant-pressure heat capacity C_p of the reservoirs is independent of temperature. Initially the reservoirs are at temperatures 300 K and 402 K. If, after some time, they come to a common final temperature T_f, the process remaining adiabatic, what is the value of T_f (in Kelvin) ? **347**

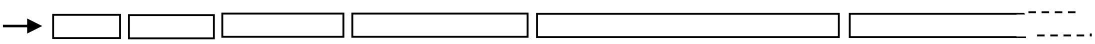
18. N particles are distributed among three energy levels having energies: 0, k_BT and $2k_BT$ respectively. If the total equilibrium energy of the system is approximately $42.5\,k_BT$ then find the value of N (to the closest integer). **100**

19. A realistic voltmeter can be modelled as an ideal voltmeter with an input resistor in parallel as shown below:



Such a realistic voltmeter, with input resistance 1 kΩ, gives a reading of 100 mV when connected to a voltage source with source resistance 50 Ω. What will a similar voltmeter, with input resistance 1 MΩ, read in mV, when connected to the same voltage source? **105**

20. An electron enters a linear accelerator with a speed v = 10 m-s^{-1} . A vertical section of the accelerator tube is shown in the figure, where the lengths of the successive sections are designed such that the electron takes the same time τ =20 ms to traverse each section.



If the momentum of the electron increases by 2% every time it crosses the narrow gap between two sections, what is the length (in km) of the collider which will be required to accelerate it to 100 km-s^{-1}? **102**

GS-2018-Y (Physics)

Section B

Q. 21 – 35 : to be attempted only by all candidates for Integrated M.Sc.-Ph.D. programme.

(Candidates for Ph.D. programme will get NO credit for attempting this section.)

PLEASE READ CAREFULLY BEFORE PROCEEDING FURTHER

The following questions (21 – 30) are all of multiple-choice type. For every question, four options (a), (b), (c) and (d) are given, of which *only one* is correct. Indicate the correct option on the OMR by filling only one bubble. If more than one bubble is filled the question will be treated as not attempted.

3 marks will be awarded for each correct answer and 1 mark will be deducted for each incorrect answer. If the question is not attempted, no marks will be awarded or deducted.

21. If $y(x)$ satisfies the differential equation

$$y'' - 4y' + 4y = 0$$

with boundary conditions $y(0) = 1$ and $y'(0) = 0$, then $y\left(-\frac{1}{2}\right) =$

(a) $-\frac{e}{2}$ (b) $\frac{1}{2}\left(e + \frac{1}{e}\right)$ (c) $\frac{1}{e}$ ✔(d) $\frac{2}{e}$

22. Given the following xy data

x	1.0	2.0	3.0	4.0	5.0
y	0.002	0.601	0.948	1.21	1.42

which of the following would be the best curve, with constant positive parameters a and b, to fit this data ?

(a) $y = ax - b$ (b) $y = a - \exp(-bx)$

✔(c) $y = a \log_{10} bx$ (d) $y = a + \exp bx$

23. The Hamiltonian of a dynamical system is equal to its total energy, provided that its Lagrangian

(a) does not contain velocity-dependent terms.

(b) has no explicit time dependence.

(c) is separable in generalized coordinates and velocities.

(d) does not have terms which explicitly depend on the coordinates.

24. A particle in a one-dimensional harmonic oscillator potential is described by a wave-function $\psi(x,t)$. If the wavefunction changes to $\psi(\lambda x,t)$ then the expectation value of kinetic energy T and the potential energy V will change, respectively, to

(a) $\lambda^2 T$ and $\lambda^2 V$

(b) T/λ^2 and $\lambda^2 V$

(c) T/λ^2 and V/λ^2

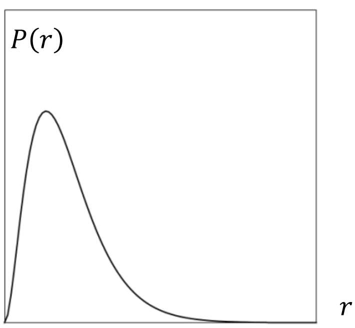
(d) $\lambda^2 T$ and V/λ^2

25. An electron is in the $2s$ level of the hydrogen atom, with the radial wave-function

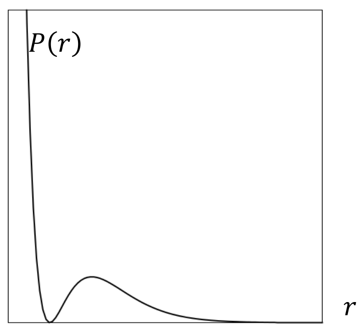
$$\psi(r) = \frac{1}{2\sqrt{2}a_0^{3/2}}\left(2-\frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right).$$

The probability $P(r)$ of finding this electron between distances r to $r+dr$ from the centre is best represented by the sketch

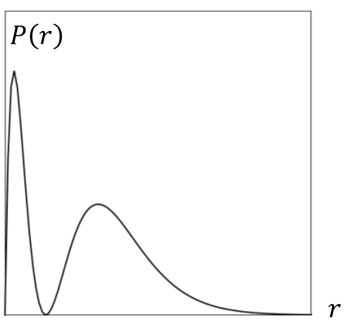
(a)



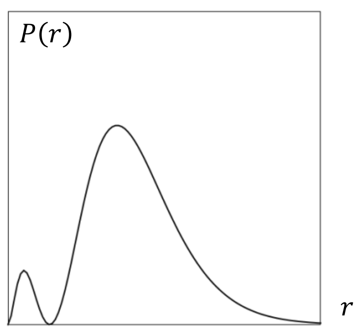
(b)



(c)



(d)



26. An atom of atomic number Z can be modelled as a point positive charge surrounded by a rigid uniformly negatively charged solid sphere of radius R. The electric polarisability α of this system is defined as

$$\alpha = \frac{p_E}{E}$$

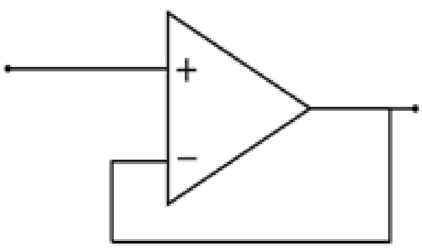
where p_E is the dipole moment induced on application of electric field E which is small compared to the binding electric field inside the atom. It follows that $\alpha =$

(a) $\frac{4\pi\epsilon_0}{R^3}$ (b) $\frac{8\pi\epsilon_0}{R^3}$ (c) $4\pi\epsilon_0 R^3$ (d) $8\pi\epsilon_0 R^3$

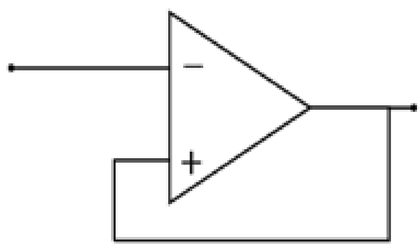
27. A many-body system undergoes a phase transition between two phases A and B at a temperature T_c . The temperature-dependent specific heat at constant volume C_V of the two phases are given by $C_V^{(A)} = aT^3 + bT$ and $C_V^{(B)} = cT^3$. Assuming negligible volume change of the system, and no latent heat generated in the phase transition, T_c is

(a) $\sqrt{\frac{4b}{c-a}}$ (b) $\sqrt{\frac{3b}{c-a}}$ (c) $\sqrt{\frac{2b}{c}}$ (d) $\sqrt{\frac{b}{c-a}}$

28. Consider the following circuits C-1 and C-2.



C-1



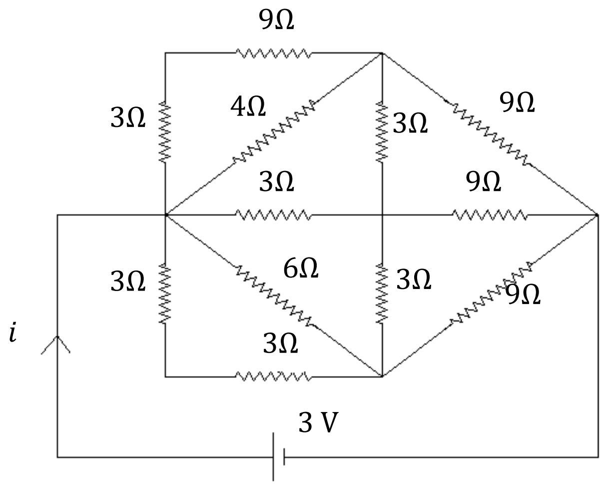
C-2

You can apply the golden rules of an ideal op-amp to

(a) both C-1 and C-2 (b) neither C-1 nor C

(c) only C-1 (d) only C-2

29. The current i flowing through the following circuit is



(a) 1.0 A ✓(b) 0.75 A (c) 0.6 A (d) 0.5 A

30. Two students A and B try to measure the time period T of a pendulum using the same stopwatch, but following two different methods. Student A measures the time taken for one oscillation, repeats this process N ($\gg 1$) times and computes the average. On the other hand, Student B just once measures the time taken for N oscillations and divides that number by N. The error in the measurement of the stopwatch is much smaller than the time period.

Assuming that the accuracy of the stopwatch is sufficiently high which of the following statements is true about the errors in T as measured by A and by B ?

(a) A and B will measure the time period with the same accuracy.

(b) It is not possible to determine if the measurement made by A or B has the larger error.

(c) The measurement made by A has a smaller error than that made by B.

✓(d) The measurement made by A has a larger error than that made by B.

Part B continues…

Part B continues…

(to be attempted by all candidates for Integrated M.Sc.-Ph.D. programme)

PLEASE READ CAREFULLY BEFORE PROCEEDING FURTHER

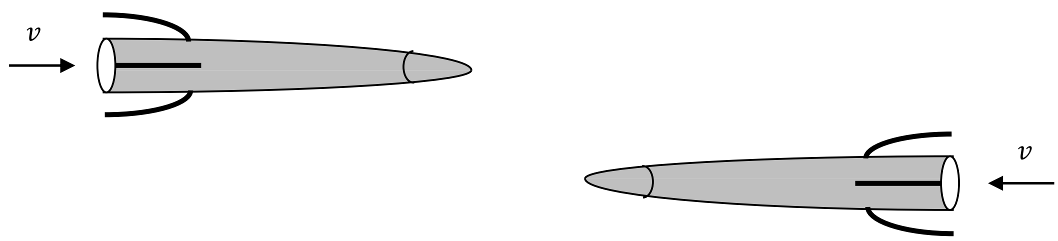
Each of the following questions (31 – 35) must be answered by a word or a number or a simple mathematical expression, which must be written down clearly and legibly using only black/blue ballpoint pen. Use only values of constants given in the table 'USEFUL CONSTANTS'.

5 marks will be awarded for each correct answer. There are NO NEGATIVE MARKS for these questions. But if there are any cancellations, overwriting or erasures, the question will be considered as not answered.

31. Evaluate the integral

$$\int_{-\infty}^{+\infty} dx \exp(-x^2) \cos\left(\sqrt{2}x\right)$$

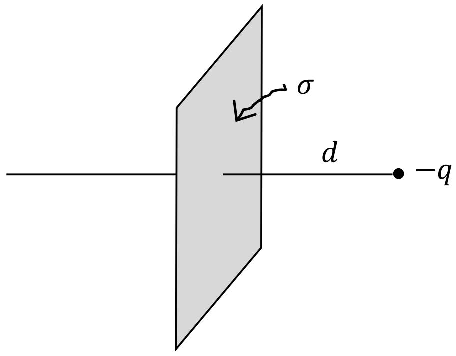
32. From an observational post E on the Earth, two ballistic missiles, each of rest length ℓ from nose-tip to tail-end, are observed to fly past each other, with the same uniform relativistic speed $c/2$, in opposite directions, as shown below.



What is the time taken for the tail-end of one of the missiles to cross the tail-end of the other missile, as measured from the post E ?

33. A statistical system, kept at a temperature T, has n discrete energy levels with equal level-spacing ε, starting from energy 0. If, now, a single particle is placed in the system what will be the mean energy of the system in the limit as $n \to \infty$? [The answer should not be left as a summation]

34. Consider an infinite plane with a uniform positive charge density σ as shown below.



A negative point charge $-q$ with mass m is held at rest at a distance d from the sheet and released. It will then undergo oscillatory motion. What is the time period of this oscillation?

[You may assume that the point charge can move freely though the charged plane without disturbing the charge density.].

35. Given a particle confined in a one-dimensional box between $x = -a$ and $x = +a$, a student attempts to find the ground state by assuming a wave-function

$$\psi(x) = \begin{cases} A(a^2 - x^2)^{3/2} & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$$

The ground state energy E_m is estimated by calculating the expectation value of energy with this trial wave-function. If E_0 is the true ground state energy, what is the ratio E_m/E_0 ?

GS-2018-Y (Physics)

Section C

Q. 36 – 45 : to be attempted only by candidates for Ph.D. programme.

(Candidates for Integrated M.Sc.-Ph.D. programme will get NO credit for attempting this section.)

PLEASE READ CAREFULLY BEFORE PROCEEDING FURTHER

The following questions (36 – 45) are all of multiple-choice type. For every question, four options (a), (b), (c) and (d) are given, of which *only one* is correct. Indicate the correct option on the OMR by filling only one bubble. If more than one bubble is filled the question will be treated as not attempted.

3 marks will be awarded for each correct answer and 1 mark will be deducted for each incorrect answer. If the question is not attempted, no marks will be awarded or deducted.

36. The Fourier series which reproduces, in the interval $0 \leq x < 1$, the function

$$f(x) = \sum_{n=-\infty}^{+\infty} \delta(x-n)$$

where n is an integer, is

✓(a) $1 + 2\cos 2\pi x + 2\cos 4\pi x + 2\cos 6\pi x + \cdots + (\text{to } \infty)$

(b) $1 + \cos \pi x + \cos 2\pi x + \cos 3\pi x + \cdots (\text{to } \infty)$

(c) $\cos \pi x + \cos 2\pi x + \cos 3\pi x + \cdots (\text{to } \infty)$

(d) $(\cos \pi x + \sin \pi x) + \frac{1}{2}(\cos 2\pi x + \sin 2\pi x) + \frac{1}{3}(\cos 3\pi x + \sin 3\pi x) + \cdots (\text{to } \infty)$

37. The value of the integral

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2}$$

is

(a) $1/2a$ (b) $1/2\pi a$ ✓(c) $\exp(-a)/a$ (d) $\pi a \exp(-a)$

38. A dynamical system with one degree of freedom is described by canonical coordinates (p, q). The generator F of the canonical transformation $(p, q) \to (-q, p)$ is

(a) $F = -p\dot{q}$ ✔(b) $F = pq$

(c) $F = p\dot{q}$ (d) $F = -\dot{p}q$

39. The electrostatic charge density $\rho(r)$ corresponding to the potential

$$\varphi(r) = \frac{q}{4\pi\epsilon_0}\frac{1}{r}\left(1 + \frac{\alpha r}{2}\right)\exp(-\alpha r)$$

is $\rho =$

(a) $q\delta(r) - 2q\alpha^3 \exp(-\alpha r)$ (b) $q\delta(r) - q\frac{\alpha^3}{4}\exp(-\alpha r)$

(c) $-q\delta(r) - 2q\alpha^3 \exp(-\alpha r)$ ✔(d) $q\delta(r) - q\frac{\alpha^3}{2}\exp(-\alpha r)$

40. The Hamiltonian of a particle of charge q and mass m in an electromagnetic field is given by

$$H = \frac{1}{2m}\left|\vec{p} - q\vec{A}(\vec{x}, t)\right|^2 + q\varphi(\vec{x}, t)$$

where $(\varphi, \vec{A})$ are the electromagnetic potentials. Clearly this Hamiltonian changes under a gauge transformation

$$\varphi \to \varphi - \frac{\partial \chi}{\partial t} \qquad \vec{A} \to \vec{A} + \vec{\nabla}\chi$$

where $\chi(\vec{x}, t)$ is a gauge function. Nevertheless the motion of the particle is not affected because

✔(a) the action of the particle changes only by surface terms which do not vary.

(b) the Lorentz force is modified to balance the effect of the gauge transformation.

(c) the Lagrangian does not change under the gauge transformation.

(d) the motion of the particle is correctly described only in the Lorenz gauge.

41. The electron of a free hydrogen atom is initially in a state with quantum numbers $n = 3$ and $\ell = 2$. It then makes an electric dipole transition to a lower energy state. Which one of the given states could it be in after the transition?

(a) $n = 3, \quad \ell = 0$

✔(b) $n = 2, \quad \ell = 1$

(c) $n = 3, \quad \ell = 1$

(d) $n = 2, \quad \ell = 2$

42. Consider a monoatomic solid lattice at a low temperature $T \ll T_D$, where T_D is the characteristic Debye temperature of the solid ($T_D = \hbar\omega_m/k_B$ where ω_m is the maximum possible frequency of the lattice vibrations). The heat capacity of the solid is proportional to

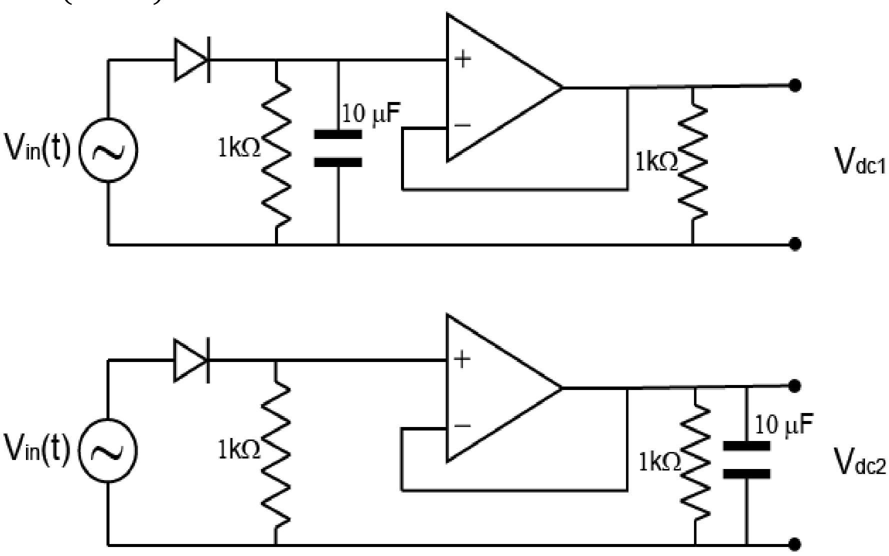
(a) T/T_D

(b) T_D/T

✔(c) $(T/T_D)^3$

(d) $(T_D/T)^2$

43. A signal $V_{in}(t) = 5\sin(100\pi t)$ is sent to both the circuits sketched below.



If the DC output voltage of the top circuit has a value V_{dc1} and the bottom circuit has a value V_{dc2}, then which of the following statements about the relative value of V_{dc1} and V_{dc2} is correct?

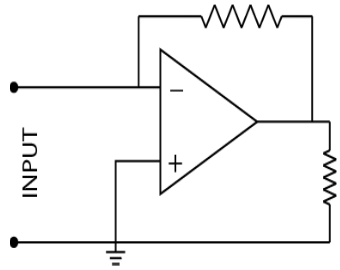
(a) It will depend on the slew rate of the op-amp.

(b) $V_{dc1} = V_{dc2}$

(c) $V_{dc1} < V_{dc2}$

✔(d) $V_{dc1} > V_{dc2}$

44. Consider the circuit shown on the right, which involves an op-amp and two resistors, with an input voltage marked INPUT.



Which of the following circuit components, when connected across the input terminals, is most likely to create a problem in the normal operation of the circuit?

(a) A voltage source with very high Thevenin resistance.

✔(b) A voltage source with a very low Thevenin resistance.

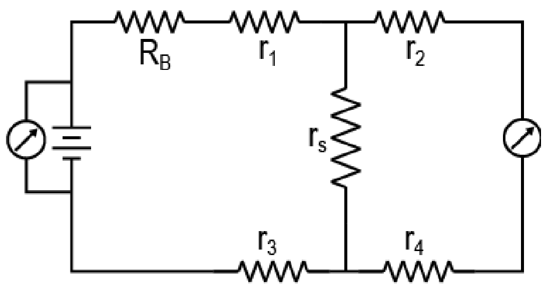
(c) A current source with a very high Norton resistance.

(d) A current source with a very low Norton resistance.

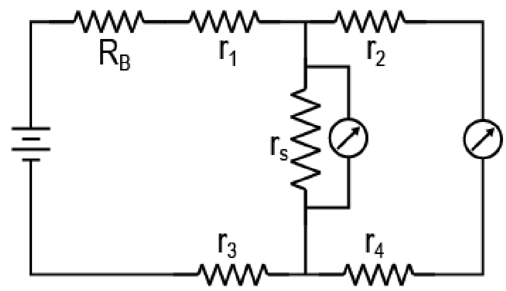
45. Which one of the following circuits, constructed only with resistors and voltmeters, will allow you to obtain the correct value of resistance r_s using the voltmeter readings? Note that the value of R_B is known while r_1, r_2, r_3 , r_4 and r_s are all unknown.

[Assume that the voltmeters and resistors are ideal.]

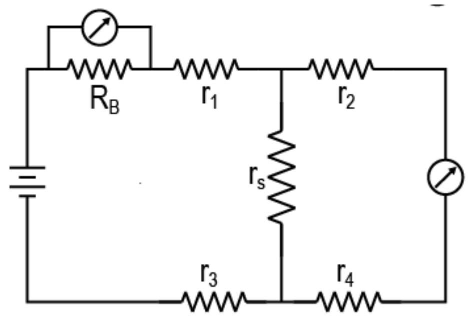
(a)



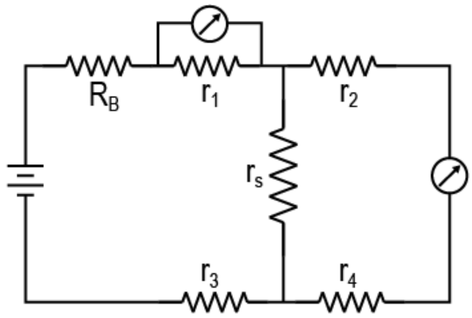
(b)



✔(c)



(d)



Part C continues...

Part C continues...
(to be attempted by all candidates for Ph.D. programme)

PLEASE READ CAREFULLY BEFORE PROCEEDING FURTHER

Each of the following questions (46 – 50) must be answered by a word or a number or a simple mathematical expression, which must be written down clearly and legibly using only black/blue ballpoint pen. Use only values of constants given in the table 'USEFUL CONSTANTS'.

5 marks will be awarded for each correct answer. There are NO NEGATIVE MARKS for these questions. But if there are any cancellations, overwriting or erasures, the question will be considered as not answered.

46. A fourth rank Cartesian tensor $T_{ijk\ell}$ satisfies the following identities

    (i) $T_{ijk\ell} = T_{jik\ell}$

    (ii) $T_{ijk\ell} = T_{ij\ell k}$

    (iii) $T_{ijk\ell} = T_{k\ell ij}$

    Assuming a space of three dimensions (i.e. $i, j, k = 1,2,3$), what is the number of independent components of $T_{ijk\ell}$?

47. If the velocity of the Earth in its orbit is v, find $\delta E/E$, where E is the translational (non-relativistic) kinetic energy of the Earth and δE is its relativistic correction to the lowest order in v/c .

48. A plane electromagnetic wave, which has an electric field

    $$\vec{E}(\vec{x},t) = (P\hat{\imath} + Q\hat{\jmath}) \exp i\omega \left(t - \frac{z}{c}\right)$$

    is passing through vacuum. Here P, Q and ω are all constants, while c is the speed of light in vacuo.

    What is the average energy flux per unit time (in SI units) crossing a unit area placed normal to the direction of propagation of this wave, in terms of the above constants ?

49. The state $|\Psi\rangle$ of a spin-1 particle is given by

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(|1,-1\rangle + |1,0\rangle \exp\frac{i\pi}{3} + |1,1\rangle \exp\frac{2i\pi}{3}\right)$$

where $|S, M_s\rangle$ denote the spin eigenstates with eigenvalues $\hbar^2 S(S+1)$ and $\hbar M_S$ respectively. Find $\langle S_x\rangle$, i.e. the expectation value of the x component of the spin.

50. A particle of mass m moves in a two-dimensional space (x, y) under the influence of a Hamiltonian

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{4}m\omega^2(5x^2 + 5y^2 + 6xy)$$

Find the ground state energy of this particle in a quantum-mechanical treatment.