GS2019: Selection Process for Mathematics

Selection process for admission in 2019 to the various programs in Mathematics in TIFR, namely the PhD and Integrated PhD programs at TIFR, Mumbai and TIFR CAM, Bengaluru, and the PhD program at ICTS, Bengaluru, is as follows:

Part I. For all the programs, the first part of the selection process will be the nationwide test conducted in various centers on December 9, 2018. This is an objective test of three hours duration, with 20 multiple choice questions and 20 true/false questions.

The score in the above test (Part I) will serve as qualification marks for a student to progress to the second step of the evaluation process. The cut-off marks for the various programs can be different.

Part II. The second part of the selection process varies according to the program and the center. More details about this part will be provided at a later date.

Syllabus for Part I

Part I of the selection process is mainly based on mathematics covered in a reasonable B.Sc. course in Mathematics. This includes:

Algebra: Definitions and examples of groups (finite and infinite, commutative and non-commutative), cyclic groups, subgroups, homomorphisms, quotients. Definitions and examples of rings and fields. Basic facts about vector spaces, matrices, determinants, ranks of linear transformations, characteristic and minimal polynomials, symmetric matrices. Integers and their basic properties. Inner products, positive definiteness.

Analysis: Basic facts about real and complex numbers, convergence of sequences and series of real and complex numbers, continuity, differentiability and Riemann integration of real valued functions defined on an interval (finite or infinite), elementary functions (polynomial functions, rational functions, exponential and log, trigonometric functions), sequences and series of functions and their different types of convergence, oridinary differential equations.

Geometry/Topology: Elementary geometric properties of common shapes and figures in 2 and 3 dimensional Euclidean spaces (e.g. triangles, circles, discs, spheres, etc.). Plane analytic geometry (= coordinate geometry) and trigonometry. Definition and basic properties of metric spaces, examples of subset Euclidean spaces (of any dimension), connectedness, compactness. Convergence in metric spaces, continuity of functions between metric spaces.

General: Pigeon-hole principle (box principle), induction, elementary properties of divisibility, elementary combinatorics (permutations and combinations, binomial coefficients), elementary probability theory, elementary reasoning with graphs.

Sample Questions for Part I

Sample multiple choice questions

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous bounded function. Then

   (a) f has to be uniformly continuous
   (b) there exists an $x \in \mathbb{R}$ such that $f(x) = x$
   (c) f can not be increasing
   (d) $\lim_{x\to\infty} f(x)$ exists.

2. Define a function

$$f(x) = \begin{cases} x + x^2\cos(\frac{\pi}{x}), & \text{if} \quad x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

   Consider the statements:

   I. f is differentiable at $x = 0$ and $f'(0) = 1$.
   II. f is differentiable everywhere and $f'(x)$ is continuous at $x = 0$.
   III. f is increasing in a neighbourhood around $x = 0$.
   IV. f is not increasing in any neighbourhood of $x = 0$.

   Which one of the following combinations of the above statements is true.

   (a) **I.** and **II**.
   (b) **I.** and **III.**
   (c) **II.** and **IV.**
   (d) **I.** and **IV.**

Sample true/false questions

1. If A and B are 3×3 matrices and A is invertible, then there exists an integer n such that $A + nB$ is invertible.

2. Let P be a degree 3 polynomial with complex coefficients such that the constant term is 2010. Then P has a root α with $|\alpha| > 10$.

3. The symmetric group S_5 consisting of permutations on 5 symbols has an element of order 6.

4. Suppose $f_n(x)$ is a sequence of continuous functions on the closed interval [0;1] converging to 0 pointwise. Then the integral

$$\int_0^1 f_n(x)dx$$

   converges to 0.

5. There are n homomorphisms from the group $\mathbb{Z}/n\mathbb{Z}$ to the additive group of rationals $\mathbb{Q}$.

6. A bounded continuous function on $\mathbb{R}$ is uniformly continuous.