GS2022 Selection Process for Mathematics

The selection process for admission in 2022 to the various programs in Mathematics at the TIFR centres – namely, the PhD and Integrated PhD programs at TIFR, Mumbai as well as the programs conducted by CAM-TIFR, Bengaluru and ICTS-TIFR, Bengaluru – will have two stages.

Stage I. A nationwide test will be conducted in various centres on December 12, 2021. Performance in this test will be used to decide whether a student progresses to the second stage of the evaluation process. The cut-off marks for a particular program will be decided by the TIFR centre handling that program. The score in the written test may also be used in Stage II.

Stage II. The second stage of the selection process varies according to the program and the centre. More details about this stage will be provided at a later date.

Syllabus for Stage I

Stage I of the selection process is mainly based on mathematics covered in a reasonable B.Sc. course. This includes:

Algebra: Definitions and examples of groups (finite and infinite, commutative and non-commutative), cyclic groups, subgroups, homomorphisms, quotients. Group actions and Sylow theorems. Definitions and examples of rings and fields. Integers, polynomial rings and their basic properties. Basic facts about vector spaces, matrices, determinants, ranks of linear transformations, characteristic and minimal polynomials, symmetric matrices. Inner products, positive definiteness.

Analysis: Basic facts about real and complex numbers, convergence of sequences and series of real and complex numbers, continuity, differentiability and Riemann integration of real valued functions defined on an interval (finite or infinite), elementary functions (polynomial functions, rational functions, exponential and log, trigonometric functions), sequences and series of functions and their different types of convergence.

Geometry/Topology: Elementary geometric properties of common shapes and figures in 2 and 3 dimensional Euclidean spaces (e.g. triangles, circles, discs, spheres, etc.). Plane analytic geometry (= coordinate geometry) and trigonometry. Definition and basic properties of metric spaces, examples of subset Euclidean spaces (of any dimension), connectedness, compactness. Convergence in metric spaces, continuity of functions between metric spaces.

General: Pigeon-hole principle (box principle), induction, elementary properties of divisibility, elementary combinatorics (permutations and combinations, binomial coefficients), elementary reasoning with graphs, elementary probability theory.

Sample Questions for Stage I

The following are some sample questions for the online test that will be held on December 12. Some question papers from previous years can be found at
http://univ.tifr.res.in/gs2022/Prev_QP/Prev_QP.htm

Sample multiple choice questions

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous bounded function. Then

   (a) f has to be uniformly continuous

   (b) there exists an $x \in \mathbb{R}$ such that $f(x) = x$

   (c) f can not be increasing

   (d) $\lim_{x\to\infty} f(x)$ exists.

2. Define a function

$$f(x) = \begin{cases} x + x^2 \cos(\frac{\pi}{x}), & \text{if} \quad x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

   Consider the statements:

   I. f is differentiable at $x = 0$ and $f'(0) = 1$.

   II. f is differentiable everywhere and $f'(x)$ is continuous at $x = 0$.

   III. f is increasing in a neighbourhood around $x = 0$.

   IV. f is not increasing in any neighbourhood of $x = 0$.

   Which one of the following combinations of the above statements is true.

   (a) **I.** and **II**.

   (b) **I.** and **III.**

   (c) **II.** and **IV.**

   (d) **I.** and **IV.**

Sample true/false questions

1. If A and B are 3×3 matrices and A is invertible, then there exists an integer n such that $A + nB$ is invertible.

2. Let P be a degree 3 polynomial with complex coefficients such that the constant term is 2010. Then P has a root α with $|\alpha| > 10$.

3. The symmetric group S_5 consisting of permutations on 5 symbols has an element of order 6.

4. Suppose $f_n(x)$ is a sequence of continuous functions on the closed interval [0,1] converging to 0 pointwise. Then the integral

$$\int_0^1 f_n(x)dx$$

converges to 0.

5. There are n homomorphisms from the group $\mathbb{Z}/n\mathbb{Z}$ to the additive group of rationals $\mathbb{Q}$.

6. A bounded continuous function on $\mathbb{R}$ is uniformly continuous.